EFFECT OF INERTIA ON THE KINETICS OF CAPILLARY UPTAKE IN A GRAVITATIONAL FIELD

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The problem of capillary uptake of a liquid taking into account inertial forces is solved. The calculation is in good agreement with experiment.

Inertial forces have a significant effect on fluid motion at the initial stage of capillary uptake, which plays an important role in the startup and switching off of different thermohydraulic capillary-porous systems. For this reason, inertial forces must also be taken into account together with other forces for the uptake process. However, taking into account these forces and gravitational forces at the same time gives rise to great difficulties in the solution of the corresponding equations of motion of the liquid in the capillary. In this paper we examine a method for obtaining an approximate solution of this equation. The results obtained are compared with experiments which we performed and the results of calculations performed by other authors.

The kinetics of liquid uptake into cylindrical capillaries under the assumption that the motion of the liquid is laminar is described by the equation [1]

$$l \frac{d^2 l}{dt^2} + \left(\frac{dl}{dt} \right)^2 + \frac{8\eta}{r^2 \rho} l \frac{dl}{dt} = \frac{2\sigma \cos \Theta}{r\rho} - gl \sin \varphi. \tag{1}$$

In most applied problems it turns out that the two terms which take into account the inertial force in this equation are much smaller in order of magnitude than the remaining terms, so that they can be neglected. The solution of this "truncated" equation is presented in [2].

For a horizontal capillary ($\varphi = 0$) and under zero-g Eq. (1) also has a quite simple analytical solution [3].

In the general case the differential equation (1) is nonlinear and its solution is obtained by approximate methods. In the first approximation the solution of this equation for a vertical capillary ($\varphi = 90°$) by the method of perturbations with respect to a small parameter, characterizing the role of the inertial force compared with other forces, is given in [4]. It is shown that when inertia is taken into account, the magnitude of the corrections to the solution of the "truncated" equation from [2] can be substantial and must be taken into account in the calculation of the kinetics of capillary uptake in a gravitational field.

The solution of Eq. (1) obtained in [4], however, is presented in an implicit form, which is inconvenient to use.

We present below a simpler approximate solution of Eq. (1), which permits describing more exactly the process of capillary uptake of the liquid at the initial stage of uptake.

As in [4], for the first initial condition for Eq. (1) assume that $l = 0$ at $t = 0$, i.e., the displacement of the meniscus equals zero initially. The second initial condition is established from the requirement that at $t = 0$ Eq. (1) must have the smallest possible singularity, in particular, the second derivative $d^2 l/dt^2 |_{t=0}$ must be finite. For Eq. (1) follows in the limit $t \to 0$, $l \to 0$ that this requirement holds only if

$$\frac{dl}{dt} = \left(\frac{2\sigma \cos \Theta}{r\rho} \right)^{1/2} \text{ at } t = 0. \tag{2}$$

For this reason, for the second initial condition for Eq. (1) we chose the condition (2).

We represent Eq. (1) in the following form:

$$\frac{d^2}{dt^2}(l^2) + a\frac{d}{dt}(l^2) + bl = c, \tag{3}$$

$$a = \frac{8\eta}{r^2\rho}; \quad b = 2g\sin\varphi; \quad c = \frac{4\sigma\cos\Theta}{r\rho}. \tag{4}$$

In this notation the initial conditions will have the form

$$l = 0, \quad \frac{dl}{dt} = \left(\frac{c}{2}\right)^{1/2} = v \quad \text{at} \quad t = 0. \tag{5}$$

We shall seek the solution of the problem (3) and (5) by the method proposed in [5, 6].

Since the functions l and l^2 satisfy Dirichlet's conditions [7], we represent them on the segment $[0, l_0]$ in the form of Fourier series

$$l = \sum_{n=1}^{\infty}\alpha_n\sin\frac{n\pi}{l_0}l, \tag{6}$$

$$l^2 = \sum_{n=1}^{\infty}\beta_n\sin\frac{n\pi}{l_0}l, \tag{7}$$

where l_0 is a constant, equal to the height of the maximum rise; in the notation (4) $l_0 = c/b$, while α_n and β_n are Fourier coefficients.

The starting equation (3), substituting (6) and (7), can be written as follows:

$$\frac{d^2}{dt^2}\sum_{n=1}^{\infty}\gamma_n\alpha_n\sin\frac{n\pi}{l_0}l + a\frac{d}{dt}\sum_{n=1}^{\infty}\gamma_n\alpha_n\sin\frac{n\pi}{l_0}l + b\sum_{n=1}^{\infty}\alpha_n\sin\frac{n\pi}{l_0}l = c\sum_{n=1}^{\infty}\frac{1}{n(n+1)}, \tag{8}$$

where

$$\gamma_n = \frac{\beta_n}{\alpha_n} = l_0\left[1 - \frac{2}{(n\pi)^2}\left(1 - \frac{1}{\cos n\pi}\right)\right].$$

Setting

$$L_n = \alpha_n\sin\frac{n\pi}{l_0}l$$

and applying formally to (8) the reduction rule [8], we write Eq. (3) with the initial conditions (5) in the form

$$\frac{d^2L_n}{dt^2} + a\frac{dL_n}{dt} + \frac{b}{\gamma_n}L_n = \frac{c}{\gamma_n}\frac{1}{n(n+1)}, \tag{9}$$

$$L_n = 0, \quad \frac{dL_n}{dt} = \frac{v}{n(n+1)} \quad \text{at} \quad t = 0. \tag{10}$$

Equation (9) for L_n is a linear differential equation, whose general solution, according to [9], is given by

$$L_n = \frac{l_0}{n(n+1)}\left\{1 - \frac{m_{1n}\exp(m_{2n}t) - m_{2n}\exp(m_{1n}t)}{m_{1n} - m_{2n}} + \frac{\dfrac{v}{l_0}[\exp(m_{2n}t) - \exp(m_{1n}t)]}{m_{1n} - m_{2n}}\right\}, \tag{11}$$

where

$$m_{1n} = -\frac{a}{2}\left(1 - \sqrt{1 - \frac{4b}{a^2\gamma_n}}\right), \quad m_{2n} = -\frac{a}{2}\left(1 + \sqrt{1 - \frac{4b}{a^2\gamma_n}}\right).$$

Taking into account (11), Eq. (6), reduced to a dimensionless form, can be written as

$$x = \frac{l}{l_0} = 1 - \sum_{n=1}^{\infty}\frac{m_{1n}\exp(m_{2n}t) - m_{2n}\exp(m_{1n}t)}{n(n+1)(m_{1n} - m_{2n})} + \frac{v/l_0[\exp(m_{2n}t) - \exp(m_{1n}t)]}{n(n+1)(m_{1n} - m_{2n})}. \tag{12}$$
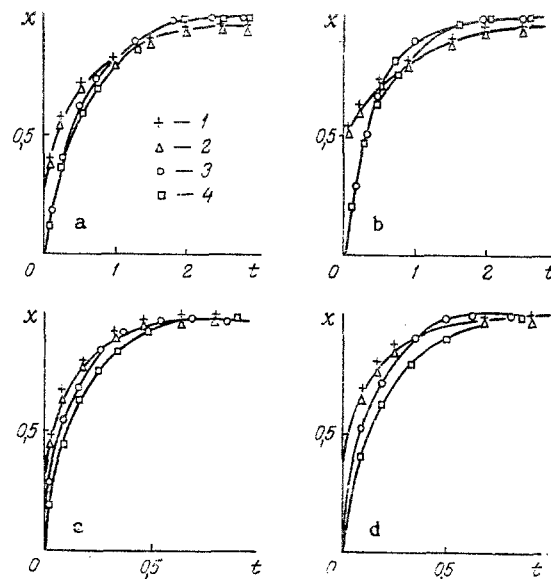
Fig. 1. Kinetics of capillary uptake of distilled water in a gravitational field: 1) calculation according to [2]; 2) [4]; 3) (13); 4) experimental data (a) $r = 0.22 \cdot 10^{-3}$ m; b) $0.24 \cdot 10^{-3}$; c) $0.28 \cdot 10^{-3}$; d) $0.32 \cdot 10^{-3}$ m). x is dimensionless; t is in sec.

In view of the weak dependence of the coefficients m_{1n} and m_{2n} on n and taking into account the fact that

$$\overline{\gamma} = \frac{1}{s} \sum_{n=1}^{s} \gamma_n \simeq l_0,$$

we represent Eq. (12) in the following approximation form:

$$x = 1 - \frac{k_1 \exp(k_2 t) - k_2 \exp(k_1 t)}{k_1 - k_2} + \frac{v/l_0 [\exp(k_2 t) - \exp(k_1 t)]}{k_1 - k_2}, \tag{13}$$

where

$$k_1 = - \frac{a}{2} \left(1 - \sqrt{1 - \frac{4b}{a^2 l_0}} \right), \quad k_2 = - \frac{a}{2} \left(1 + \sqrt{1 - \frac{4b}{a^2 l_0}} \right).$$

To check the reliability of the formula (13) we studied experimentally the capillary uptake of a liquid. The capillaries were fastened in a strictly vertical position on a holder with a measuring scale drawn on it. The liquid was distilled water, which was fed to the bottom edge of the capillary with the help of a system of connected vessels. When the water level approached the inlet into the capillary, a Konvas-avtomat motion-picture apparatus, equipped with a built-in timer, was switched on. Photographs were made on the average for 3-5 sec at a rate of 40 frames per second. For the experiments we used glass capillaries, which were carefully washed and dried at low pressure (P ~ 1330 N/m²). This preparation of the surface ensured that contaminants were removed from the walls of the capillary, which was confirmed by the good reproducibility of the experimental results on the measurement of the maximum height to which the water rose in the capillary. The investigations were carried out for cylindrical capillaries with different diameters at room temperature.

The results of the experiments and the computed data for the capillaries studied with different radii are presented in Fig. 1. It is evident that for the capillaries studied the formula (13) describes more accurately than do the fomrulas from [2] and [4] the kinetics of the process when inertial forces significantly affect the motion of the liquid.

NOTATION

l, displacement of the meniscus; t, displacement time of the meniscus; r, radius of the capillary; φ, angle of inclination of the capillary axis to the horizontal; ρ, η, σ, density, viscosity, and surface tension of the liquids; and θ, wetting angle.

LITERATURE CITED

1. A. P. Porkhaev, Kolloidn. Zh., 11, No. 5, 346-353 (1949).
2. R. A. Martynov, V. V. Malev, and E. V. Gribanova, Kolloidn. Zh., 45, No. 2, 245-250 (1983).
3. R. Siegel, J. Appl. Mech., 28, No. 2, 165-170 (1961).
4. A. I. Moshinskii, Kolloidn. Zh., 46, No. 2, 279-283 (1984).
5. G. A. Surkov, Inzh.-Fiz. Zh., 36, No. 2, 286-289 (1980).
6. G. A. Surkov and V. I. Yurinok, Inzh.-Fiz. Zh., 42, No. 1, 119-122 (1982).
7. G. M. Fikhtengol'ts, Course in Differential and Integral Calculus [in Russian], Vol. 3, Moscow (1962), pp. 414-461.
8. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Moscow (1977), pp. 44-45.
9. L. É. Él'sgol'ts, Differential Equations [in Russian], Moscow (1957), pp. 112-124.

EFFECT OF TEMPERATURE, STORAGE TIMES, AND STRESS STATE ON THE CAKING OF BULK MATERIALS

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Results are reported from experimental studies of the susceptibility of bulk materials to caking in relation to temperature conditions, storage times, and the stress state of the material.

Several investigations [1, 2] have evaluated the suceptibility of finely dispersed bulk materials to caking by adopting a physical quantity τ_0 which characterizes the initial unit resistance to shear with stress removed, i.e., at $\sigma = 0$.

The cakeability of finely dispersed bulk materials is affected by the storage time B, the moisture content of the materialW, and the prestress state σ_{con} of the material [1-5].

However, it must be noted that the consolidation process is also significantly affected by temperature conditions and the granulometric and granulomorphological composition of the material and external factors such as vibration and aeration.

The regulation GOST 21560.4-76–21560.5-76 [3] is used to evaluate the cakeability of fertilizers. The regulation is used to determine the strength of briquettes produced in special molds under certain conditions. Methods based on crushing of specimens were proposed in [4, 5] to determine the causes of consolidation. These methods can be used only for highly compacted materials such as mineral fertilizers, in which agglomerates are formed.

Bulk materials do not always form strong agglomerates in the food, chemical, mining, and other industries. Thus, the initial unit resistance to shear τ_0 is used to determine the factors which affect consolidation.

We propose to study the effect on cakeability of the ambient temperature T (K) and storage time B (days), with a fixed stress state σ_{con}, fixed initial moisture W, and fixed granulometric and granulomorphological composition d_i. We studied both inorganic and organic materials: kaolin, concentrated at W = 0.60% and d_a = 11.70 µm; chalk at W = 0.5% and d_a = 9.87 µm; flour of grade 1 at W = 13.00% and d_a = 45 µm; wheat groats at W = 10.75% and d_a = 750 µm (more details on the characteristics of the materials are given in [8]). The studies were conducted at an ambient temperature t = 22 ± 1°C and a relative humidity of 75%.

Results of determination of the cakeability of the above materials as a function of temperature T (258-308 K) and storage time B (from 1 to 5 days), with a constant prestress σ_{con}= 30 kPa and an initial moisture content W = const, are shown in Fig. 1. It can be seen from